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In light of the psychological factors discussed, the following suggestions are made for the set-up of a mathematics program for socially disadvantaged children. In pre-school, children should be subjected to perceptual training and concept development relative to space, number, time and related conceptual domains, since these later become the basis for conceptual systems symbolically represented. The curriculum should be analyzed and altered with respect to the sequential relatedness of the subject matter, the demands on this group of students, and the testing procedures used. Techniques should be used which will insure the orderly transition to symbolic representation. The concepts represented should provide a challenge for the child. Supplementary materials which are self-instructional should be provided. Problems of teacher recruitment and retention should be recognized. Less concern should exist about imposing middle class culture on lower-class children.

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Psychological issues in the development of mathematics curricula for socially disadvantaged children

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This paper is a revision of an invited address to the Conference on Mathematics Education for Below Average Achievers held by the School Mathematics Study Group in Chicago, April 10-11, 1964.

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Psychological issues in the development of mathematics curricula for socially disadvantaged children[1]

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In characterizing the learning problems of socially disadvantaged youth some well-meaning critics are of the opinion that all persons are in some way disadvantaged either by lack of intelligence, social status or limitation in personality. These critics hold that a response to one group's disadvantage and not those of all groups represents a policy which is at once unfair and educationally unsound. Although there may be something to recommend this view, it fails to recognize crucial distinctions among the "disadvantaged." Thereas a white middle-class child may differ from his fellows in intelligence or wealth, he is not without sufficient personal and social resources to overcome his particular "disadvantage." This luxury is not afforded the lower-class youth who very early in life loses the opportunity to develop the means which, at the minimum, would enable him to adapt to the economic and social life of the community. The most unfortunate feature of slum experience is that by the time adolescence is reached children are so limited in intellectual and social skills as to make a personal reversal of their fate extremely difficult. It, therefore, behooves us to strain our resources to provide such youngsters with the skills which will make an economically and personally

gratifying life possible consistent with both a moral imperative and the enlightened self-interest of the community.

Class differences in achievement

The differences in intellectual development and scholastic achievement between urban poor and advantaged children are no myth. By the time the average slum child reaches the seventh or eight grade he is approximately two to three years behind his middle-class counterpart in reading skill. In New York City junior high schools classes are homogeneously grouped by reading ability. In one such school located in a slum area the average reading grade achievement score for one eighth grade class was 6.7; another eighth grade class (not the lowest) in the same school had an average reading score somewhat above the third grade level (3.4). These average figures mask the seriousness of the problem since such values only indirectly indicate that a substantial number of children are not even able to read at the first grade level. The assessed intellectual performance of slum children is likely to be at least 10 to 15 IQ points lower, on the average, than that of other children, and scholastic achievement is likely to be retarded in all areas of academic learning. (The average IQ for the former eighth grade class cited above was 89.4; for the latter 70.6). Interestingly enough, the areas of intellectual performance which show the least difference in level of performance are those associated with mathematical reasoning (Rosner, 1956).[2] Factor analysis of intelligence test performance, on the other hand, indicates that the types of reasoning

processes (as distinct from levels of performance) employed by both middle-class and lower-class children are the same. The social class differences in intelligence and achievement among seventh graders are generally greater than those of children who first arrive at school, but the differences even at the early grades are striking. What is particularly distressing, moreover, is that performance differences in the early grades are in "tool" skills, that is, in the skills that enable the child to process formal information offered in the school curriculum and informal information provided by his out-of-school experience. The urban slum child, whether Negro, Puerto Rican, Mexican, Indian or white has lived five or six years in an environment which both physically and socially has placed severe constraints upon certain features of his development. This is not true for all aspects of his development, for the rigors of this existence force certain kinds of social adaptation upon the child that makes him more resilient and vigorous than some of the hot house varieties bred elsewhere in urbia and suburbia.

Constraints imposed by slum experience--language deficits

On the other hand, the lower-class home is limited in that the actual intellectual and social interchange between parents and children is of a kind that does not provide a practical and easy transfer to the school setting. In the lower-class home, for example, meal times do not provide opportunities to acquire and test one's language skills. Reading periods are practically non-existent. The most significant outcome of this is the very

limited language facility of the slum child (Milner, 1951). Not only is there limited content in the language, but lower-class life both in the home and in the community fosters the development of a linguistic "code" which is different from that of members of the middle class. One investigator (Bernstein, 1961, 1962) describes lower-class language as following a "restricted" linguistic code pattern while middle class environments develop both "elaborative" as well as restrictive codes. Furthermore, the restrictive code of the middle class is different from that of the lower class. The middle-class restrictive code is usually of a special or formal language (e.g., mathematics), whereas the restrictive code of the lower class might be, for example, the "hip" language of some lower-class adolescents. The importance of these different orientations to language is that they are not equally suited to school instruction. Bernstein further suggests that a limited linguistic code hinders the development of an extensive functional vocabulary and keeps thought at a low level of conceptualization.

Even if lower-class parents desire to play an active role in the education of their children they are limited by their own lack of language and knowledge. On the whole, lower-class parents are far less effective as teachers than middle-class parents (Hess, 1964). Many lower-class parents act as though they subscribe to the view that if a child does not give the correct answer to a problem, the best way to get it out of him is to threaten or use physical coercion.

The lower-class parent is not only far less effective as a directive agent for learning, but is unable to provide the child with an environment adequate for learning. Books are not present and read in the home; nor is it a very convenient place to bring books. Children may be kept awake very late by a blaring television set. They cannot be expected to have a place to study. The relative language isolation of the children is aggravated by missing fathers, frequent geographic dislocation, poor diets, crowded space, and a high incidence of disease.

Other cognitive deficits

Limited language resources are not the only legacy of lower-class life. The lower-class child arrives at school with a wide variety of cognitive and precognitive deficits. He is less capable, for example, of making perceptual discriminations among physical objects in his environment, less able to deal with the pictorial representation of objects and actions, and more limited in the ability to conceptualize in even primitive ways. In one of our studies with Kindergarten children there were consistent social class differences in the judgment of equal and unequal areas, as well as in the ability to equate areas when their configurations were altered (Beilin, 1964). The tasks given these children were designed to determine whether: 1. two squares (subdivided into four units each) were equal in area, 2. a square (of four units) was equal or unequal to a figure with five or three units, and 3. a square was equal in area to other figures (also with four units)

rearranged so that the patterns were not square. In another study we also found differences between lower and middle-class second and third grade children in the ability to imagine what the water level would be in covered jars tilted in various orientations to the horizontal and in pictorial representations of these jars (Beilin and Kagan, in prep.). Almy has likewise found consistent differences between middle and lower-class New York children in the logical capacities to deal with the "conservation" of number and volume problems described by Piaget. In these problems the child is asked to count the number of blocks he sees (lined in a row before him.) The same blocks are then spread out (in the same line). The child paradoxically insists there are now more blocks, even if he recounts them and recognizes they are the same number (Almy and Chittenden, 1963).

Non-cognitive problems

The urban slum child who arrives at the school with these limited cognitive resources is bound to find his experience frustrating unless the school is prepared to overcome these cognitive deficits. It would be naive, however, to assume difficulties reside solely in the cognitive and intellectual realms. Those who prepare curricula for disadvantaged children need to recognize that these children arrive in the classroom with other characteristics which inhibit learning. The disordered life in which the slum child grows does not, in general, foster the development of those internalized controls that keep a child attending to a task for protracted

periods of time or enable him to attend to what the teacher is saying and doing. These children are much more easily distracted than their peers and much more likely to act out their wants in physical ways. Their requests are more often transmitted through action than words, and are therefore often perceived as hostile and aggressive. Such children characteristically nudge or touch the teacher to get her attention. They take the ruler another child is using rather than ask for it. The ruler is just as likely to be used as an extension of the arm in a fencing match as an extension of the arm for measurement. The order and self-discipline required of the learner in school settings is alien to the experience of many of the children we are describing. This kind of self-discipline, however, is as necessary to learning as are cognitive skills since teaching may be delayed until some semblance of personal and group order is achieved.

One is likely to find, too, that materials, devices and language which are new and different hold little attraction. The child may be intrigued and stimulated by the initial novelty of the school but interest will progressively wane, even more than with other children, unless something more sustaining than novelty is introduced. If novelty is not followed by some success with the newly encountered materials, the frustrations resulting from unsuccessfull attempts at mastery will result in a retreat from the potentially interesting school environment.

Learning and Developmental processes

In considering the possibilities for reversing the effects of the slum child's experience, it is necessary to examine the learning and developmental processes which have brought him to the point of being seen as both socially and intellectually disadvantaged.

The work of Piaget on the development of cognitive functions suggests invariance in the order of development of such functions (Flavell, 1963). He offers evidence for a series of developmental stages in which the achievements of a particular period are dependent upon those of prior periods. The data suggest that if the experience of a child is restricted, both in character and extent, the child's intellectual resources may be expected to be limited also. Although there is controversy over whether the sequence in cognitive development is altered by experience, what Piaget and others seem to be in agreement about is that the transitions from one stage of intellectual development to the next <u>are</u> influenced by various kinds of experience.

The earliest developments so affected are of perceptual-motor operations. The first responses to sensory experience are motor. In the crib, for example, the child's first reaction to rattles and similar objects is to accidentally strike them and by so doing discover they produce interesting sounds. These actions then tend to be repeated, and the coordination between what he sees and what he does becomes "represented" in his muscle and neural systems. Through what are at first unintentional actions he learns to

coordinate his activity with his sensations, rapidly acquiring a repertoire that makes possible later intentional action. It is through these primitive sensory-motor "schema" that the child is first able to construct his reality. From this kind of direct action there is gradual transformation of direct experience to "images" of action. Whereas the child at first has need to see a toy to know that it exists he gradually comes to the recognition that it exists even when not seen, as when hidden behind a screen or placed under a pillow. The manipulation of this imagery permits vicarious problem solving and elementary conceptualization. Symbolic representation begins first in the imaginal representation of action--then develops through language and pictorial representation. The ability to deal with one's experience through the imaginal representation of action permits the child much greater scope in dealing with reality than through action alone and provides the base for more intelligent behavior. The acquisition of language permits even greater extension of the child's intellectual capacities, for it provides him not only with content for his thought but with symbolic vehicles for thought. Although the young child (prior to age six) is endowed with language and imagery these have not fully liberated his thought from inferences made directly from the way stimuli are organized. As Piaget has illustrated (Wohlwill, 1962) the young child's response to reality is determined largely by perceptual processes that focus on single physical variables. For example, the number of objects put before him becomes greater if the objects are

spread out, water volume increases if spread thin, areas are made larger if they are partitioned and rearranged. By school age, however, the normal child develops the cognitive resources that enable him, in spite of perceptual incongruities and ambiguities, to attain invariant conceptions of "physical" concepts (area, number, volume). Through these capacities (which Piaget denotes as "conservation" abilities) the child is able to reason that the number of objects before him remain the same even if placed in different locations, and volumes remain invariant irrespective of the shapes of the vessels into which they are poured. In this era of the child's development, which lasts till about pre-adolescence, the capacities for classificatory and relational thinking are developed. These intellectual abilities are limited, however, by the dependence of the child's thought upon the manipulation of concrete objects and their pictorial or schematic representation. It is not till early adolescence that the child's thought is liberated from the manipulation of concrete materials, and he is able to deal with elements and relations in a wholly abstract and symbolic fashion. The child becomes capable, in fact, of thinking in formal propositional terms.

To put the Piaget developmental scheme more succinctly there are three general periods in the child's cognitive development: sensory-motor operations, concrete operations and formal operations. In general outline the sequence of development may be fairly fixed although this is far from established. Even if the sequence is

fixed, the age at which a particular child makes the transition from one stage to the next is not fixed. The variability in making these transitions is, in fact, considerable.

Evidence is available to indicate that the performance of urban slum children is markedly inferior to that of middle-class' children in these cognitive advancements (Almy and Chittenden, 1963). In the stage of concrete operations when the "conservation" capacities of children normally reach fruition Almy found a 25% difference between the ability of slum and middle-class children (first and second graders) to perform correctly on three conservation tasks.

"Readiness"

A momentary diversion may be in order. In speaking of stages, as we have, and the ability and inability to function at particular periods in the child's life it may seem that we might suggest certain guidelines for when a child is "ready" to learn. This notion of "readiness" has been considerably abused in recent times. Many people have been justifiably exercised over the idea that children have been kept from learning how to read till they were six because they were not "ready." It is very true that in educational circles this notion has for years had the force of law. It has been made clear of late, however, that as a generalization this makes little sense. In turn, the notion of readiness itself has been vigorously attacked. This is a good instance of the baby being thrown out with the bath water. What should have been specifically attacked was that the evidence for the 6 year "readiness rule" was flimsy

at best. It was the scientific evidence for this generalization which was at fault, not the general evidence with regard to "readiness." The idea of readiness can be approached more fruitfully in a rather different way. Even some investigators who are convinced that children can be taught to learn almost any concept provided it is presented in a manner congruent with their stage of development (Bruner, 1960) place at least implied limitation on this generalization. Bruner, for example, cites a study (by Frank) in which children at two ages (4 and 5) are provided with verbal experience in solving a "conservation" problem (Bruner, 1964). The older children are able to use the verbal information; the younger are not. In another study it was likewise shown that it was not possible (albeit in a limited demonstration) to teach qualitative area measurement to first graders, but it was possible to teach qualitative length measurement to them (Beilin and Franklin, 1962). There <u>are</u> limitations upon when and how children can learn. Our task is to discover what these limitations are and under what conditions they operate and with whom. The danger is in calling a halt too soon to efforts to discover what is possible in the child's learning. Nevertheless, it is just as foolish to say all children are prepared to learn at a particular age as it is to say no children are prepared to learn.

<u>Cognitive representation</u>

In an attempt to view cognitive development from a somewhat different orientation, Bruner places greater emphasis on the processing of the data of experience. He suggest three ways in

experience is represented in the child's thought. The first type of representation is enactive, i.e., it results from motor activity. The experience of learning to ride a bicycle gets represented kinesthetically. Verbalization is not likely to help another to learn this skill. The second type of representation is iconic: it refers to internal images of what is visually, tactually or auditorily perceived. The final type of representation is symbolic. In this type the events of experience are represented in either natural or formal languages. The nature of development is such that one style of dealing with the world is successively supplemented by more abstract and hence more encompassing modes of representation. The scope of the child's ability to act upon the world, as well as his capacity to be acted upon, is considerably enhanced by the more abstract and symbolic types of representation, but these are the most difficult of achievement.

The work of Krutetski is illuminating here (Krutetski, 1963). In a study of Russian children identified as having an "incapacity for mathematics" Krutetski identified two general components of intellectual activity as basic to mathematics learning: a "visual-image" component and a "verbal-logical" component. In testing the interrelation of these two factors he identifies four groups of children who suffer from an incapacity for mathematics. What these groups of children have in common is the inability to solve mathematics problems posed in a verbal-logical form. (For example, "There are two circles with a radius of 2 cm. and 3 cm. The distance between their centers is 10 cm. Do these circles intersect?") The ability to

deal with visual-image representations of the problems is not a consistent failing, i.e., there are children who do solve visual-image problems and fail with the verbal-logical representations, but they do badly in general with mathematics learning. Unfortunately little is reported of the "backgrounds" of these poor learners so that the origins of these verbal-logical deficits is not clear. What does emerge from the Krutetski, Piaget, and other studies is that there is a pronounced relationship between the ability to perform well in mathematics and the ability to deal with the symbolic and abstract representations of problems, whether verbally or nonverbally. Socially disadvantaged children are particularly deficient in the ability to represent their experience symbolically and this circumscribes their potentialities for learning mathematics.

From the viewpoint of both Piaget and Bruner, the crucial issue in cognitive development is the transition from one stage of development to the next--or from one form of cognitive representation to another. Although it has been possible for psychologists to identify and characterize the stages in cognitive development, it has not been possible to explain very satisfactorily how the transitions from one stage to the next are achieved. The mechanisms of change are rather obscure. This makes the task of the educator particularly difficult if he desires to accelerate the transitions when a child is retarded or slow.

The slum child, upon arrival at school, is more characteristically an "enactive" child than is the middle-class child. He is also less able than the middle-class child to deal with concrete materials, and much less with their pictorial or schematic representations. For example, when given blocks to sort according to particular characteristics (e.g., size, color, form) or when given pictures of objects or line drawing representations of them, the slum child will have greater difficulty recognizing the objects or performing operations with them. In essence, he will have more difficulty with concepts related to objects and their symbolic representation than with objects themselves. The disparities between children become greater as the middle-class child acquires greater competence in the symbolic realm, while the lower-class youngster continues to struggle with concrete materials.

Transitions and translations

The principal task for work with disadvantaged children, as we see it, is to facilitate the transition from one level of thinking to the next through the development of special instructional techniques and materials. Children who are particularly "action oriented" or "enactive" will profit from the translation of motor activity to reasoning with perceptual (visual, auditory and tactual) types of representation. Those who demonstrate facility at the perceptual level, in turn, require a translation to the symbolic level, particularly in the special language with which we are concerned, the language of mathematics. For example, a child who is able to sort out large

from small blocks may be able to learn to do the same task when presented with pictures of blocks, or of houses. From this basis he may be taught to associate the words "large" and "small" with each of the groups. The child thus may be moved progressively to the point of being able to deal with the logical relationship between large and small verbally (symbolically) without direct reference to the physical objects. Another level of organization and difficulty can be learned by introducing 3 blocks or houses of different size with the objective of coming to recognize that the middle object is both smaller than the larger and larger than the smaller and becoming able to verbalize it as well. The child may be trained to carry the understanding of this relationship to other objects, as well as to other kinds of relations such as "long"-"short," "wide"-"narrow," etc. The development of translational techniques (i.e., of learning the equivalence of a concept represented visually and verbally) requires a prerequisite knowledge of the methods of instruction with each type of representation. The most difficult of the tasks, however, relates to the translational techniques themselves through which reasoning in relation to one form of representation is made equivalent to reasoning in another. These techniques are not altogether new to mathematics instruction. Some materials, such as the Cuisenaire and Montessori rods, already do this. From a deliberate use of lengths of wood divided by color into equal units one moves to symbolically representing and manipulating the numerical relations among the "rods."

An illustration of the Montessori approach is seen with the "counting beads." When ten individual beads have been obtained from a receptacle by the child they are exchanged for a string of ten beads. Ten such strings may then be put together in a square frame. Ten of these squares can be piled up and shown to be equivalent to a cube of 1000 beads. In this manner children physically handle sets of ten, sets of 100, and sets of 1000 with the units observable within the sets.

Montessori methods have been built, in large measure, on the notion of translation from sensory experience to the symbolic. It should be recalled that Montessori did not begin her work with the middle-class children who typically attend Montessori type schools now, but with "street urchins" in the San Lorenzo slum section of Rome. The basic techniques for which she became known were developed with them. As instructors who use these and similar materials recognize, once facility is developed in use of the formal language system, the manipulation of the concrete materials may and should be set aside. An exception to this is made when children are prepared to attain a higher order process. A preparatory period is introduced in which manipulative materials are temporarily used to facilitate the transition to the higher order problem.

The use of concrete materials to provide a basis for learning abstract relations is appropriate not only at the level of learning "larger and smaller" and computation in a 10 based number system. The approach may be applied at more advanced levels, as in the use of a balance in learning quadratic equations. Dienes' work is designed, in part, to establish isomorphisms between the logical organizations

among concrete materials and the same relationships represented symbolically (Dienes, 1963).

The most important of the translations to be made are those that bring the child from reasoning "concretely" to his reasoning symbolically. But even reasoning with the use of concrete materials (as in numerical relations among blocks, or the beads of an abacus) may require a great deal of training for disadvantaged children before a translation can be made into language equivalents and to formal notation. It should be noted that much of the success of the so called "new" mathematics has been demonstrated with middle-class children and the ability of young children to utilize sophisticated notational systems has yet to be demonstrated successfully with disadvantaged children.

Cognitive efficiency

Generally, neither the child nor adult functions wholly at the highest level of intellectual performance of which he is capable. He employs the kind of thought process most efficient for the demands of a problem and this may or may not be the most sophisticated thought process available to him. In one of the aforementioned studies (Beilin, 1964) it was found that children used an iterative measurement method to establish the equality between 4 unit area patterns that were differently arranged, but used a translocational method (i.e., ideationally moving the units to other positions) when the patterns contained 9 units, that is, when the patterns contained too many units for the children to easily count. This suggests that

two kinds of information should be sought for every cognitive learning experience: first, the kinds of logical methods that may be used by the learner to solve a problem; second, the appropriate manner of presenting problems in order to elicit particular logical solution methods. In this way one will ensure that the child is able to master a maximum of solution strategies.

The disadvantaged as slow learners

Some educators of disadvantaged youth with whom we have discussed these issues have characterized disadvantaged children as slow learners and advise that instruction for them should proceed with the techniques developed for slow learners. This position tends to overlook some crucial distinctions. Among the disadvantaged is a type of learner of limited genetic endowment, whom the best of instruction would profit in only limited ways. Many disadvantaged children are slow learners in a different sense. Their progress can be much more rapid with appropriate techniques of instruction. A study by Revoire (Jensen, 1963) demonstrates the differential capacities of two groups of children with low IQs, one white and one Mexican. The latter group profited from the introduction of verbal materials in their learning; the former group did not. It was reasoned that these particular Mexican children represented a socially disadvantaged group whose performance provided only a limited view of their potential intellectual status. This suggests that under the proper circumstances the introduction

of verbal techniques of instruction may foster rapid learning with some disadvantaged children, whereas they might not with children who are slow learners because of mental retardation.[3] To put it another way, the optimal techniques of translation for one group of slow learners may be different from those of another. Needless to say, very little has been done in experimenting with these techniques.

The age of instruction

Since so many of the problems of learning with the socially disadvantaged start at a very early age, it would be desirable to initiate programs related to mathematics instruction at the earliest school age possible; even in the kindergarten and nursery school. In preschool classes, pre-mathematics programs should emphasize perceptual training and concept development relative to space, number, time and related conceptual domains since these later become the basis for the development of conceptual systems symbolically represented. The development of curricula should proceed from the early grades upward.

Redundancy and the "education of attention"

We have indicated earlier that an important difference between the lower and middle-class child is the greater distractability and limited attention span of the former. It has been implied by some researchers that conceptual learning with young learners requires building into their training a maximum of "redundancy" (Wohlwill, 1962)

Reduncancy can take two forms. Either material is repeatedly presented with no new information added, or else the same concepts are presented in new contexts or new forms with little or no new information added. According to the "redundancy" view a greater number of encounters with the learning materials is required for young and slow learners to acquire a concept. All who have worked with disadvantaged children know this to be true for them. Older, brighter, and advantaged children require less redundancy. There is a danger, however, in merely repeating the same materials ad nauseum in the hope a child will learn. It is the danger of boredom and frustration for both the pupil and teacher. Fashioning multiple examples of the same concept may provide the opportunity, however, for such learning. Dienes and others (e.g., Churchill, 1961) take this position in suggesting a variety of instances for representing a concept should be offered the learner. Dienes' intent is to provide the child with a variety of representations of the same concept in order that the child may learn to abstract the concept from its many instances, but it might also be argued that the many representations of the concept merely provide the child with greater opportunity to encounter and thereby process the information necessary to develop a concept (whether by a process of abstraction or by some other process).

Coupled with the need for "repetition" is the need for the units of instruction to be smaller, since slow learners in general have difficulty in remembering or retaining information even very

recently acquired. In the language of information theory these children have rather limited channel capacity and overloading them will only break down the communication system between teacher and learner.

Having presented this view of concept learning we wish to offer another which is presently receiving considerable attention among psychologists. It suggests that in perceptual and cognitive learning the crucial determinant in learning is the "education of attention" (Gibson, 1963). The child learns to attend to the critically differentiating details, progressively eliminating a concern for the irrelevant. The objective of teaching would be the identification of the relevant dimensions of difference and similarity in the attainment of concepts. The need for redundancy is minimized as the relevant details become more salient. This approach is evidenced in the ways in which children may learn the alphabet with an "alphabet board" made of a display panel with alphabet cutouts into which the child may fit wooden letters. Attempting to fit the letters into the board draws the child's attention to the details of form and contour. Preschool children become progressively more adept at discriminating and identifying the letters through this sensory-motor training. This kind of training has relevance for first grade learning to read since there are data to indicate that there are significant differences between middle and lower-class Negro children in knowledge of the alphabet upon entering the first grade (Weiner and Feldman, 1963).

Curriculum philosophy

There have been two philosophies of mathematics instruction which have divided practitioners in their work with children. These views are being carried over to work with the disadvantaged. One group which has its origins in the Froebelian tradition places its emphasis upon the attainment of mathematical principles through varied experience. The child is presented with the opportunity to manipulate meaningful materials. In response to the problems encountered with these the child develops intellectual skills. At one end of the continuum, this practical experience may take the form of dealing with the problems of paying and receiving in a store (Churchill, 1961), or at the other end, with the manipulations of a balance (Dienes, 1963).

The extent of external manipulation of the learner's experience depends upon the orientation of the teacher, but the intent is for principles and generalizations to be abstracted or "discovered" from experience. Teachers of the disadvantaged who work in this tradition say they can make use of linguistic and other modes of response already available to the child, even if these do not fit into accustomed patterns. This approach, they hold, permits the development of greater intellectual fluency and has the advantage of utilizing the child's prior experience.

The alternative view is oriented to providing the learner with a logical structure or grammar which he may apply to a variety of physical elements and relations. Here the materials are developed in a logically ordered fashion so that the child acquires fundamental

logical principles in a sequential order gradually developing more complex and sophisticated skills. At one end of the continuum, one may have the sequential programs of Montessori (Montessori, 1964) starting with sensory experience to, at the other end, the deductive strategies of those who derive theorems and provide the opportunity to apply them to varieties of problems. It seems reasonable to us that the disadvantaged child, who is lacking in most cases in relevant cognitive structures or the symbolic means of representing his experience, may profit from being provided with appropriate logical models or problem solving strategies. He may thus be helped to bypass the often arduous and frustrating task of attempting to discover principles which come more easily to middle-class children (with their considerably broader referential systems).

At this stage in our knowledge of mathematical learning it is not possible to adjudicate the difference between these views. A decision as to which strategy will result in greater returns in learning is an empirical question which will require more research and exploration

Curriculum suggestions

From these many observations we come to the point of considering specific suggestions for mathematics curriculum development.

One curriculum specialist (Bristow, 1964), in reflecting on curriculum proposals for reducing school dropouts, cited a number of curriculum decisions which had been accepted in a variety of

quarters: 1. Elimination of the practice of failing students, 2. meeting individual differences (in classes, schools and interests), 3. rejection of homogeneous grouping, 4. elimination of tracking systems, 5. a refusal to accept logically arranged materials, and 6. a rejection of school grade organization. He noted, in each instance where this had been done, that no program had been adequately thought through or had been experimented with in anything resembling a systematic procedure. He noted further that, whether the alternatives were wise or not, curriculum development had resulted more from the "insights" and experience of teachers and supervisors who had worked with the disadvantaged than upon conceptually based programs of curriculum development or research.

There are, then, no thought-through curriculum strategies available for working with disadvantaged children. What this adds up to is that groups like the School Mathematics Study Group and others who anticipate preparing curricula for disadvantaged children have an immense opportunity and responsibility in addressing themselves to the problem. The opportunity is that there presently is no competition; the responsibility, that a mediocre job might prove immensely successful in comparison to present practice.

While we have no major strategy to offer we do have some suggestions. In offering these suggestions we assume that the major subject matter decisions will have been made. That is, that the goals of mathematics instruction reflected in the concepts and skills to be learned will have been specified.

One caveat seems in order. A pilot project of the Biological Sciences Curriculum Study (Grobman, 1964) similarly specified their goals in developing special materials for the "750,000 tenth-graders who either take a special slow-learner course in biology or who are assigned to a rather elementary type of special 'general science' course." The BSCS pilot study was concerned with testing the feasibility of an approach to teaching complex biological concepts to slow learners. Many of the classes designated as slow-learner classes, however, had aptitude levels considerably higher than those found in classes of socially disadvantaged children. Two important guidelines set for the special programs were, first, that the program should stress laboratory work, and second, that the reading level and length of reading assignments should be appropriate to the reading level of the students. Even though the researchers evaluated the pilot work as encouraging and are convinced of the feasibility of their approach, it should be pointed out that the average post-test achievement for one of the learning units (across all of the classes) was only 55%. While the results may indicate more learning than is usually found in such classes, it seems important to emphasize that with 55% success instructional problems were far from solved.

Suggestion 1: Begin below the first grade. Some of the "newer" curriculum revisions have started at the high school level and worked down the grades. While expediency and fortuitous circumstance may lead to entering the curriculum in the middle years of schooling,

it is less likely that such an approach will lead to a coherent mathematics curriculum for disadvantaged children.

Mathematics instruction for the disadvantaged should include skills and concepts usually assumed to have been learned prior to the first grade. One has to ensure that children can make the necessary perceptual discriminations (e.g., of form) upon which other concepts will be dependent (e.g., size, order). In turn, these concepts will lead to the abstract relations that are independent of particular physical events or objects and that are the subject matter of mathematics.

Suggestion 2: Subject the curriculum to a behavioral and cognitive analysis. The strategy we propose involves three steps: 1) a conceptual analysis of the mathematical ideas and skills to be learned, i.e., an analysis of the logical relatedness of mathematical concepts; 2) an analysis of the cognitive learning and response systems required of the learner, and 3) empirical testing involving the relation between the two. These procedures require careful specification of what is to be taught and what resources the learner has available to learn.

In testing instructional sequences with children from disadvantaged backgrounds, a number of researchers have discovered that they have assumed a variety of behaviors and capacities that were in fact missing to the children. Many of these behaviors were ones that they did not even realize were required for learning nor were many of these easy to identify or remedy (Gotkin and Goldstein, 1964).

Resnick, for example, describes the procedures used in revising a programed instructional sequence on the grid coordinate system to make it suitable for culturally deprived children. After five revisions the program still could not be considered successful since 40% of the target population did not learn (Resnick, 1963). Attempting such an adaptation, however, demonstrated how this strategy could help identify some simple but significant behaviors which inhibit learning.

We have found it valuable to conduct the empirical testing with individual children. In group testing of learning sequences one or two children are usually able to offer the correct responses and thereby mask the difficulties encountered by other children. When we are able, we have an experimental psychologist, a master teacher, a subject specialist, and a research assistant present for such observations. The research assistant or the master teacher presents the material to the individual learner while the others observe. The value of this approach was demonstrated when we found that some children could not respond to what seemed simple questions. Rewording the questions produced strikingly different results.

Suggestion 3: <u>Utilize or develop techniques which will ensure the orderly transition to symbolic representation</u>.

This embodies our most important point with regard to working with disadvantaged children.

One should be prepared to take the learner where one finds him but anticipate with the preparation of adequate materials and techniques to move as rapidly as possible to learning mathematics on a symbolic level. The stress with materials prepared for the early years should be on sensory-motor manipulation. Later materials may be pictorial and still later, materials may be abstract and symbolic. The presentation of materials may be either simultaneous or sequential depending on the learning status of the child.

Techniques should be oriented to making evident to the child the isomorphism between the representation of the concept in visual form and its representation in symbolic form. Further, sufficient experience should be provided with the two forms until the child is able to proceed with some assurance using solely the symbolic. Instructional units should: A) Be limited in size. B) Provide for a variety of ways of representing a concept. C) Provide for different levels of difficulty within a conceptual scheme.

Suggestion 4: Materials should be developed so that the concepts represented provide a challenge for the child.

Problems should be presented the child that represent some cognitive uncertainty (Smedslund, 1964). It should be an uncertainty, however, that can be reduced by the utilization of previously acquired learning. This is a departure from approaches which place great stress on the reinforcement of appropriately made responses. Although we see considerable value in approriate "drill" procedures

(particularly with mathematical "facts") we feel the focus of the program should be on training that will maximize the child's active solution of problems and to some extent the self-generation of the strategies of solution. Again, this should not preclude providing verbal or visual models where it is felt that these will give greater mileage to the child's problem solving ability.

Suggestion 5: Supplementary materials should be provided that are self-instructional.

We have emphasized that slum children live in homes that are not conducive to studying and that their schools are characteristically overcrowded. Also, these children are geographically mobile: Reports of 100% turnover in a classroom within a school year are not uncommon. Teachers are less likely to be well-trained and are themselves more transient. It is necessary, if one is concerned with developing effective curricula to take these limitations into consideration. It is necessary to construct curriculum materials that do not depend on the active participation of parents in the home. The materials must also be suitable for school settings that are minimally organized, and must also be flexible enough so that students who enter in the middle of the term can cope with them.

For these reasons it is suggested that some supplementary materials be as self-instructional as possible. One may look to the technology of programed instruction for guidance in developing these materials, but it is necessary at the same time to recognize

that developing the orientation to work with such materials is difficult even for non-slum children.

Suggestion 6: Recognize the problems of teacher recruitment and retention.

In schools serving lower-class children up to 25 per cent of teaching time is provided by teachers with emergency substitute licenses (Sexton, 1961). Teaching time provided by emergency substitutes is four times greater in these schools than in schools serving middle-class populations. It is essential to recognize, of course, that unlicensed teachers may be very fine teachers and even well-trained, despite their not having met particular licensing requirements. Yet it would be very difficult to make a case for the number of teachers who teach mathematics who have not met the standards represented by such a subject matter license.

From Table 1 it is evident that in a major city like New York the number of teachers not licensed to teach mathematics and yet teaching it is very considerable. This problem is more acute with mathematics than with any other area of academic instruction. At that, the situation in New York may be better than in many other areas of the country.

The matter of lacking adequate training in mathematics is aggravated by the attitudes of teachers working in disadvantaged schools. There are many competent, hardworking and dedicated teachers who take on the difficult task of working with slum children. Many of these teachers feel, however, that subject matter learning holds a low priority for them. For some, the prime

Table 1

Number and Percent of Vacancies for Authorized Positions in Mathematics by School Level, 1962

School Level	Number of Authorized Positions	Number of Teachers without permanent (Math) Licenses	Percent of Teachers without permanent (Math) Licenses
Academic High School	722	286	40
Vocational High School	93	63	68
Junior High School	1,350	980	73
Elementary School	60	51	85

Source: New York City Board of Education Report (1962)

goal is to contribute to the slum child's social adjustment; to his "getting along." The emphasis is on the development of social skills, most other skills being less important. There are other teachers who feel slum children will not go to college and the best education for them is vocational. As a consequence, less intellectual effort has been expended on them, and less intellectual effort is expected of them. Others are so disillusioned by the need to spend a large portion of time on classroom management problems that they feel the task of teaching is hop less.

There are, of course, objectively good reasons for many of these feelings. The schools many of these teachers work in were physically qualified for demolition a generation ago. The overcrowding is notorious: 40 children in a classroom is not unusual. Compliance withstate laws controlling the size of classes are avoided through various subterfuges. Classroom management is a constant source of concern, with physical violence not infrequent. Taken together with the fact the teacher may have only a rudimentary knowledge of her subject matter, the difficulties are obvious. It should be remembered, too, that the elementary school teacher has to be equally knowledgeable about Science, Social Studies (Geography, History, etc.), the so-called Language Arts (reading, writing, etc.) as well as be proficient to some degree in Art and Music instruction.

No matter how excellent the mathematics curricula are that are developed they will be paper programs only if no concern exists for the people who will have to use them and the conditions that prevail in the classrooms in which they will be used. The "newer" mathematics curriculum developers have of course provided special services to teachers, but the training problems with teachers who will work with disadvantaged children will be even greater than for teachers in middle-class schools.

Suggestion 7: <u>One should not be put off by those who hold that the school middle-class culture should not be imposed upon lower-class children</u>.

It is very true that the school is a middle-class institution and its teachers reflect it. For good or for bad many of the school's practices and ideals reflect a generally middle-class orientation. The slum child, his parents, and his school face two possibilities, however, either the child acquires the skills and knowledge to make his way in a world requiring "middle-class skills" or by design or default does not (assuming the community even gives him the opportunity for a choice between the alternatives). The ways of making a decent living without acquisition of these basic skills are rapidly being reduced. Abolishing intelligence testing, or talking "hip" talk with the kids will not do it. Providing them with an adequate educational opportunity for the kind of education that will enable them to acquire skills is what may keep them from poverty and may direct them toward a life with some prospect for self-

fulfillment. It need not do it through alienating him from his family or his culture.

A two-fold opportunity exists--one for him--and one for us.

Footnotes

1 This is a revision of an invited address to the Conference on Mathematics Education for Below Average Achievers held by the School Mathematics Study Group in Chicago, April 10-11, 1964.

2 This may be an artifact, however, of a more restricted range of performance in mathematical reasoning than in other intellectual areas, or of the greater dependence on verbal or reading ability in the other areas.

3 As was brought out in the discussion of this paper there are learners who may be characterized as "slow" who are able to achieve the same levels of performance as "normal" or "fast" learners but need more time to do so. We are not referring here to his group but to those learners who need augmented or special instruction to achieve better.

References

Almy, M. and Chittenden, E. Young children's thinking: understanding of the principle of conservation. Presented at the Biennial meeting of the Society for Research in Child Development, Berkeley, 1963.

Beilin, H. Perceptual-cognitive conflict in the development of an invariant area concept. J. exper. child Psychol. (In Press).

Beilin, H. and Franklin, I. C. Logical operations in area and length measurement: age and training effects. Child Developm., 1962, 33, 607-618.

Beilin, H. and Kagan, J. Water level problem performance as a function of verbal and nonverbal training. (In preparation).

Bernstein, B. B. Social class and linguistic development. In Halsey, A.H. and Floud, J. and Anderson, C.A. (Eds.) Education, Economy, and Society. Free Press, 1961.

Bernstein, B. B. social class, linguistic codes and grammatical elements. Language and Speech, 1962, 5, 221-140.

Bristow, W. H. Curriculum problems of special import for early school leavers. In Schreiber, D. (Ed.) The School Drop-out. Washington: NEA, 1964.

Bruner, J. S. The process of education. Cambridge: Harvard Univ. Press, 1960.

Bruner, J. S. The course of cognitive growth. Amer. Psychol., 1964, 19, 1-15.

Churchill, E. M. Counting and measuring. Toronto: Univ. of Toronto Press, 1961.

Dienes, Z. P. An experimental study of mathematics learning. London: Hutchinson, 1963.

Flavell, J. H. The developmental psychology of Jean Piaget. New York: Van Nostrand, 1963.

Gibson, E. J. Perceptual development. In Stevenson, H.W. (Ed.) Child psychology. Yearbook, National Soc. Study Educ., 1963.

Gotkin, L. G. and Goldstein, L. S. Programed instruction in the schools: innovation and innovator. In Miles, M.B. Innovation in education. New York Teachers Coll. Bur. of Public., 1964.

Grobman, H. A. report on the evaluation program of the BSCS. The BSCS.-SM (slow learner) program--a pilot study. Paper presented at Amer. Educ. Res. Assoc. Meetings, Chicago, Feb., 1964.

Hess, R. D. Educability and rehabilitation: future of the welfare class. Paper presented to Committee on Human Development. U. of Chicago, April 15, 1964.

Jensen, A. R. Learning in the preschool years. J. nursery Educ., 1963, (Jan.), 133-139.

Krutetski, V. A. Some characteristics of the thinking of pupils with little capacity for mathematics. In Simon, B. & J. (Eds.) Educational Psychology in the U.S.S.R. Stanford Univ. Press, 1963.

Milner, E. Study of the relationship between reading readiness in grade school children and pattern of parent-child interaction. Child Developm., 1951, 22, 95-112.

Montessori, M. The Montessori method. Cambridge: Robert Bentley, 1964.

New York City Board of Education, Division of Personnel, Bureau of Appointments. Condition of day school organizations depicting authorized position and personnel condition by division, rank, subject and license as of October 21, 1962. (statistical data).

Resnick, Lauren B. Use of programed instructional materials with culturally deprived children. Technical Report 630521. The Center for Programed Instruction, June, 1963.

Rosner, B. Community socioeconomic status in mental organization. Unpublished doctoral dissertation, Teachers College, Columbia University, 1955.

Sexton, P. C. Education and income. New York: Viking Press, 1961.

Smedslund, J. Educational psychology. In Farnsworth, P. Annual Review of Psychology. Palo Alto: Annual Reviews, 1964.

Weiner, M. and Feldman, S. Measurement of Reading skills in lower socioeconomic status children. Paper read at Amer. Psychol. Assn. Convention, Phila.,

Wohlwill, J. F. From perception to inference: a dimension of cognitive development. In Kessen, W. and Kuhlman, C. (Eds.) Thought in the young child Monogr., Soc. Res. Child Developm., 1962, 27, 87-107.